Calculus Warm Up #11-1

Evaluate the Integrals:

1) $\int \frac{4}{x^2+9}\,dx$ 2) $\int \frac{4x}{x^2+9}\,dx$ 3) $\int \frac{4x^2}{x^2+9}\,dx$

HW Questions: p. 506

27. $\int x \sec^2 x \, dx$

$u = x$ $dv = \sec^2 x \, dx$

du / dx

$v = \tan x$

$= x \tan x - \int \tan x \, dx$

$= x \tan + \ln |\cos x| + C$

31. $\int \arctan x \, dx$

39)

41) → $\int x^2 e^{2x}\,dx$

	$u \downarrow$	$dv \uparrow$
$+$	x^2	e^{2x}
$-$	$2x$	$\frac{1}{2}e^{2x}$
$+$	2	$\frac{1}{4}e^{2x}$
$-$	0	$\frac{1}{8}e^{2x}$

43)

63) $\int_0^1 e^{-x}\sin\pi x\,dx$ $\quad u = \sin\pi x$, $du = \pi\cos\pi x$ $\quad dv = e^{-x}dx$, $V = -e^{-x}$

du & v same

$= -e^{-x}\sin\pi x - \int -e^{-x}(\pi\cos\pi x)\,dx$

$= -e^{-x}\sin\pi x + \pi\int e^{-x}\cos\pi x\,dx$ $\quad u = \cos\pi x$, $du = -\pi\sin\pi x\,dx$

$= -e^{-x}\sin\pi x + \pi\left[-e^{-x}\cos\pi x - \int -e^{-x}(-\pi\sin\pi x)\,dx\right]$

$= -e^{-x}\sin\pi x + \pi\left[-e^{-x}\cos\pi x - \pi\int e^{-x}\sin\pi x\,dx\right]$

$\int e^{-x}\sin\pi x\,dx = -e^{-x}\sin\pi x - \pi e^{-x}\cos\pi x - \pi^2\int e^{-x}\sin\pi x\,dx$

$+\pi^2 e^{-x}\sin\pi x\,dx \qquad +\pi^2\int e^{-x}\sin\pi x\,dx$

$$\frac{(1+\pi^2)\int e^{-x}\sin\pi x\,dx}{(1+\pi^2)} = \frac{-e^{-x}\sin\pi x - \pi e^{-x}\cos\pi x}{(1+\pi^2)}$$

$$\int_0^1 e^{-x}\sin\pi x\,dx = -\frac{1}{1+\pi^2}\left[e^{-x}\sin\pi x - \pi e^{-x}\cos\pi x\right]_0^1$$

9.3 Trigonometric Integrals using the power rule.

For:

$$\int \sin^a x \cos^b x \, dx \quad \& \quad \int \sec^a x \tan^b x \, dx$$

When a & $b \geq 2$

Example:

$$\int \sin^3 x \cos^4 x \, dx$$

* Integration by parts results in an infinite loop so we need a new tool!

Other tools we will need for sine & cosine:

Pythagorean Identity:

$$\sin^2 x + \cos^2 x = 1$$

Half Angle Formulas:

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

Example:

$\int \sin^3 x \underbrace{\cos^4 x}_{u^4} \, dx$

* To use the Power Rule, du must be present.

Let u = cos x

du = - sin x dx

$-\int \underbrace{\sin^2 x}_{\downarrow} \underbrace{\cos^4 x}_{u^4} \underbrace{(-\sin x \, dx)}_{du}$

$-\int (1 - \cos^2 x) \cos^4 x (-\sin x \, dx)$

$-\int (\cos^4 x - \cos^6 x)(-\sin x \, dx)$

$(u^4 - u^6)(du)$

$-\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + C$

$\int \sin^2 x \cos^5 x \, dx$

Let u = sin x

du = cos x dx

$\int \underbrace{\sin^2 x}_{u^2} \underbrace{\cos^4 x}_{\text{rename}} \underbrace{\cos x \, dx}_{du}$

Rename

$\cos^4 x$

$(\cos^2 x)^2$

$(1 - \sin^2 x)^2$

$1 - 2\sin^2 x + \sin^4 x$

$\int \sin^2 x (1 - 2\sin^2 x + \sin^4 x)(\cos x \, dx)$

$\int (\sin^2 x - 2\sin^4 x + \sin^6 x)(\cos x \, dx)$

$\frac{\sin^3 x}{3} - \frac{2\sin^5 x}{5} + \frac{\sin^7 x}{7} + C$

$\int \cos^2 x \, dx$

If $u = \cos x$

$du = -\sin x \, dx$

$\frac{1}{2}\int (1 + \cos 2x)\, dx$

$\frac{1}{2}\left[x + \frac{1}{2}\int 2\cos 2x \, dx\right]$

$u = 2x$
$du = 2$

$\frac{1}{2}x + \frac{1}{4}\sin 2x + C$

$\int \cos^2 x \, dx$

If $u = \cos x$

$du = -\sin x \, dx$

If only one trig word (sin or cos),
or if they are both even powered,

use the half angle identities.

Tools for secant and tangent:

Pythagorean Identity: $1 + \tan^2 x = \sec^2 x$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \sec^4 x \tan x \, dx$$

Let $u = \sec x$

$du = \sec x \tan x \, dx$

$$\int \sec^3 x (\sec x \tan x \, dx)$$

$$\frac{\sec^4 x}{4} + C$$

Ex: Find the area under the curve bounded by $y = \tan^4 x$ and the x-axis on $[0, \frac{\pi}{4}]$

$$\int_0^{\frac{\pi}{4}} \tan^4 x \, dx$$

Let $u = \tan x$

$du = \sec^2 x \, dx$

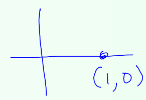
$$\int \tan^2 x \cdot \tan^2 x \, dx$$

$$\int \tan^2 x (\sec^2 x - 1) dx$$

$$\int \underbrace{\tan^2 x}_{u^2} \underbrace{\sec^2 x \, dx}_{du} - \int \tan^2 x \, dx$$

$$\frac{\tan^3 x}{3} - \int (\sec^2 x - 1) \, dx$$

$$\left. \frac{\tan^3 x}{3} - \tan x + x \right]_0^{\pi/4}$$



$$\frac{1}{3} - 1 + \frac{\pi}{4} - (0 - 0 + 0)$$

$$\boxed{\frac{\pi}{4} - \frac{2}{3}}$$

HW: p. 516, # 1 - 7 odd, 17, 24, 25, 65

HW Quiz Tuesday:

pgs. 456, 497, 506 first assignment only